Université de Paris 7, site de Chevaleret - 175 rue du Chevaleret, Paris 13 ème.

Mardi 30 mars 2010, de 16h00 à 17h00 - salle a determiner

Pietro Dello Stritto

Titre: Buildings and Moufang polygons whose first order theory is supersimple of finite rank.

Abstract: *Buildings* are certain combinatorial and geometric structures originally introduced by J. Tits (around 1955) as an attempt to find a meeting point between projective geometry and the theory of simple Lie groups. The easiest examples of buildings come from flag complexes of a projective space over a (skew) field K; among these we have the buildings of type $A_l(K)$, $l > 1$, with those of rank 2 being nothing but projective planes *PG(2,K)* whose associated *little projective group* is $PSL_3(K)$. In [6], Tits classified all *spherical*, *irreducible* buildings of rank at least 3. The rank 2 residues of the latter turn out to be exactly *generalized n-polygons*, or *n-gons*, for short. The classification of generalized polygons is not currently possible, and therefore one needs a stronger condition arising from the group action, called the *Moufang condition*, in order to classify them; it is a strong homogeneity condition for buildings. Under this assumption Moufang polygons are classifiable, and a complete list is given by Tits and Weiss in [7]. It was shown by Tits that if spherical, irreducible buildings of rank at least 3 are *thick*, then they are already Moufang, and that all rank 2 residues are Moufang too; vice versa, Tits also showed that these buildings are an amalgamation, in a certain precise sense, of some Moufang generalized polygons.

We assume familiarity with the model-theoretic notion of (*super*)*simple* first order theories. However, even if one is not familiar with this notion, for *this* talk a quick glance through [4] and [Chapter 5, 8] would suffice. We will give a brief introduction to the subjects above, motivate our model-theoretical approach, and mainly discuss Moufang polygons, but also mention buildings of higher rank, whose first order theory is supersimple of finite rank. The idea comes from [2], where the authors classified Moufang polygons under the superstable finite Morley rank assumption. We will compare and discuss the differences. The finite Morley rank condition is extremely strong, and eliminates many interesting Moufang generalized polygons; for example, those associated with twisted simple groups. Some of the latter, instead, do enter the picture under the supersimplicity assumption (for instance, Moufang octagons).

Our main result is the following, where by *good* Moufang polygons we mean, up to *duality*, those families of Moufang polygons which include arbitrary large finite ones; namely, families whose members are either *Desarguesian projective planes*, *symplectic quadrangles*, *Hermitian quadrangles* in projective space of dimension 3 or 4, *split Cayley hexagons*, *twisted triality hexagons* or *Ree-Tits octagons*, with the latter arising over a *difference* field (a field equipped with an automorphism).

Theorem: Let $M = M(K)$ be a supersimple Moufang polygon of finite rank. Then:

- *(i)* the (difference) field K is definable in M;
- *(ii)* M is a good Moufang polygon.

Examples of good Moufang polygons whose first order theory is supersimple of finite rank come quite naturally from the finite, although this requires some work to be done: basically, by developing the asymptotic theory of the families of finite Moufang polygons, one can show that they form *asymptotic classes of finite structures*, in the sense of [3]; this is done in [1], and makes strong use of

techniques from [5]. It then follows, immediately, that non-principal ultraproducts of these asymptotic classes give rise to examples of *measurable* (notion introduced in [3]) Moufang polygons and, therefore, supersimple finite rank ones.

Thus, most of the work is left with the other direction: to eliminate *bad* Moufang polygons. Basically, there is an algebraic structure *S* (e. g. an alternative division ring, a Jordan division algebra, and so on) and a (skew) field *K* associated to *S* which "determine", up to duality, the associated generalized polygon, and vice versa. In the language of polygons (point-line incidence) the field *K* is "visible", and the key point is to define it. Given a bad Moufang polygon it is not always possible to define the associated field *K*, but, however, "some" field is definable anyway, as well as "something" which is not compatible with supersimplicity; for instance, as occurs in most of the cases, sometimes we can define a non-surjective norm map from a finite degree extension of the *defined* (difference) field, which contradicts the main theorem from [4].

REFERENCE:

[1] P. Dello Stritto, "Asymptotic classes of finite Moufang polygons", submitted to Journal of Algebra. Preprint version available at:
www.logique.jussieu.fr/modnet/Publications/Preprint%20server/papers/206/index.php

[2] L. Kramer, K. Tent, H. van Maldeghem, "Simple groups of finite Morley rank and Tits buildings", Israel Journal of Mathematics XX (1999) 1-36.

[3] H. D. Macpherson, C. Steinhorn, "One-dimensional asymptotic classes of finite structures", Trans. Amer. Math. Soc. 360 (2008) 411-448.

[4] A. Pillay, T. Scanlon, F. O. Wagner, "Supersimple fields and division rings", Math. Research Letters 5 (1998) 473-483.

[5] M. Ryten, PhD Thesis: "Model theory of finite difference fields and simple groups", University of Leeds (2007)
http://www.logique.jussien.fr/modnet/publications/preprint/20server/papers/123/123.pdf

[6] J. Tits, *Buildings of spherical type and finite BN-pairs*, Lectures Notes in Mathematics 386 Springer, Berlin, 1974 - Second edition, 1986.

[7] J. Tits, R. M. Weiss, *Moufang polygons*, Springer Monographs in Mathematics, 2002.

[8] F. O. Wagner, *Simple theories*, Kluwer Academic Publishers, vol. 503, 2000.