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DEFINABLE AUTOMORPHISMS OF $\mathscr{P}(\omega)/\mathrm{fin}$

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ABSTRACT. We investigate definable automorphisms of $\mathscr{P}(\omega)/\mathrm{fin}$ and show that e.g. every Borel automorphism is trivial. The existence of nontrivial projective automorphisms is consistent and independent from ZFC + CH.

Throughout this paper $\mathscr{P}(\omega)/\mathrm{fin}$ will denote the Boolean algebra of $\mathscr{P}(\omega)$ modulo the ideal of finite sets. Call an automorphism F of $\mathscr{P}(\omega)/\mathrm{fin}$ trivial if there are two cofinite sets $a, b \subseteq \omega$ and a bijection $f\colon a \to b$ such that $F[x] = [f''(a \cap x)]$. Here $[x]$ denotes the equivalence class of x under the equivalence relation $x =_* y$ iff $(x \setminus y) \cup (y \setminus x)$ is finite. Note that there are exactly $2^{\aleph_0}$ trivial automorphisms. It is a well-known fact, first proved by W. Rudin [7], that the Continuum Hypothesis implies there are $2^{2^{\aleph_0}}$ automorphisms of $\mathscr{P}(\omega)/\mathrm{fin}$, hence most of them are nontrivial. It is also consistent with $\neg$CH that there is a nontrivial automorphism of $\mathscr{P}(\omega)/\mathrm{fin}$. This was proved by Baumgartner (unpublished). He noticed that if there is a family $\{a_\alpha\colon \alpha < \omega_1\}$ of subsets of ω such that

(i) $a_\alpha \setminus a_\beta$ is finite for every $\alpha < \beta < \omega_1$,

(ii) $\{a_\alpha\colon \alpha < \omega_1\} \cup [\omega]^{<\omega}$ generates a maximal ideal I,

then one can build by induction a family $\{f_\alpha\colon \alpha < \omega_1\}$ of functions such that

(iii) $f_\alpha\colon a_\alpha \to a_\alpha$ is a permutation without fixed points,

(iv) if $\alpha < \beta$ then $f_\beta \restriction a_\alpha =_* f_\alpha$.

Then we simply define $F\colon \mathscr{P}(\omega)/\mathrm{fin} \to \mathscr{P}(\omega)/\mathrm{fin}$ by

$$F[x] = \begin{cases} [f_\alpha''(x)] & \text{if } x \in I \text{ and } x \subseteq a_\alpha \text{ for some } \alpha < \omega_1, \\ [\omega \setminus f_\alpha''(\omega \setminus x)] & \text{if } x \notin I \text{ and } \omega \setminus x \subseteq a_\alpha \text{ for some } \alpha < \omega_1. \end{cases}$$

Now, (iv) ensures that F is well defined, (ii) that it is defined everywhere and (iii) that it is nontrivial. To see that, assume F is induced by some $f\colon a \underset{\text{onto}}{\overset{1\text{-}1}{\to}} b$ for $a, b \subseteq \omega$ cofinite. One can easily find disjoint infinite sets a^0, a^1 and a^2 such that $a^0 \cup a^1 \cup a^2 = a$ and $f(a^\varepsilon) \cap a^\varepsilon = \varnothing$ for $\varepsilon = 0, 1, 2$. Let $\varepsilon < 3$ be such that $a^\varepsilon \notin I$. It follows from the definition of F that $f''(a^\varepsilon) \notin I$, but, since I is a maximal ideal, $f''(a^\varepsilon)$ and a^ε must intersect which is a contradiction.

The existence of a family satisfying (i) and (ii) is consistent with $\neg$CH (see Kunen [3, p. 289]).

Using the method of Laver [4] we can prove the consistency of "ZFC + MA(σ-centered) + $\neg$CH + there is a nontrivial automorphism of $\mathscr{P}(\omega)/\mathrm{fin}$". It is

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not known to the author if MA + $\neg$CH implies, or is at least consistent with, the existence of nontrivial automorphisms.

On the other hand, S. Shelah [9] proved the following.

THEOREM 1. *If* ZF *is consistent then so is* "ZFC + $2^{\aleph_0} = \aleph_2$ + *every automorphism of* $\mathcal{P}(\omega)$/fin *is trivial*".

Here, we shall be interested in definable nontrivial automorphisms of $\mathcal{P}(\omega)$/fin. Throughout the rest of the paper we identify $\mathcal{P}(\omega)$ and 2^ω, i.e. $x \subseteq \omega$ is identified with its characteristic function χ_x. For a pointclass Γ we say that F is a Γ-automorphism if $\mathrm{Gr}(F) \in \Gamma$, where $\mathrm{Gr}(F) = \{\langle x, y\rangle: F[x] = [y]\}$. The following lemma is a distilled form of Shelah's Main Lemma from [9, p. 131].

LEMMA 2. *Let* $F \in \mathrm{Aut}(\mathcal{P}(\omega)/\mathrm{fin})$. *Suppose there is a dense* G_δ *subset* X *of* 2^ω *and a continuous* φ: $X \to 2^\omega$ *such that* $\forall x \in X\, F[x] = [\varphi(x)]$. *Then* F *is trivial.*

PROOF. Let us first assume $X = 2^\omega$. For $s \in 2^{<\omega}$ let $N_s = \{x \in 2^\omega: s \subseteq x\}$ be the basic clopen set determined by s. For $s, t \in 2^{<\omega}$ say that s *forces* t if $\varphi(N_s) \subseteq N_t$. We build by induction an increasing sequence of integers $\langle n_i: i < \omega\rangle$ and functions f_i: $[n_i, n_{i+1}) \to 2$ such that

(i) for every $i < \omega$ and every $s \in 2^{n_i}$, $s \cup f_i$ forces some $t \in 2^{n_i}$,

(ii) for every $i < \omega$, $s, s' \in 2^{n_i}$, every $k > n_{i+1}$ and g: $[n_{i+1}, k) \to 2$ if $s \cup f_i \cup g$ forces some t and $s' \cup f_i \cup g$ forces some t' then $\forall i \geqslant n_{i+1}\ t(i) = t'(i)$ if both are defined.

Assume f_{i-1} and n_i are defined. We have to construct f_i and n_{i+1}. First, we take care of (i).

Enumerate 2^{n_i} as $\{s_j: j < 2^{n_i}\}$ and construct partial functions f_i^j for $j \leqslant 2^{n_i}$ by induction. Set $f_i^0 = f_{i-1}$. Suppose we have f_i^j and it satisfies $\mathrm{dom}(f_i^j) \cap n_i = \varnothing$. Since φ is continuous we can find $f_i^{j+1} \supseteq f_i^j$ such that $s_j \cup f_i^{j+1}$ is a function which forces some $t \in 2^{n_i}$. Finally let $g_i = f_i^{2^{n_i}}$. It is clear that if $f \supseteq g_i$ is such that $\mathrm{dom}(f) \cap n_i = \varnothing$ then f satisfies (i) in place of f_i.

To take care of (ii) we need an auxiliary definition. For $s, s' \in 2^{n_i}$ call a partial function f a *witness* for $\langle s, s'\rangle$ if there is some $n > n_i$ such that

(iii) f: $[n_i, n) \to 2$,

(iv) whenever $k > n$ and g: $[n, k) \to 2$, t, t' are in $2^{<\omega}$ such that $s \cup f \cup g$ forces t and $s' \cup f \cup g$ forces t', then t and t' agree above n, i.e. if

$$\sigma = (\mathrm{dom}\, t \cap \mathrm{dom}\, t') \setminus n$$

then $t \restriction \sigma = t' \restriction \sigma$.

Claim. For every partial function τ: $[n_i, k) \to 2$ *and every* $s, s' \in 2^{n_i}$ *there is a partial function* $f \supseteq \tau$ *which is a witness for* $\langle s, s'\rangle$.

PROOF. Assume otherwise. We can produce sequences of partial functions $\langle \tau^j: j < \omega\rangle$, $\langle t^j: j < \omega\rangle$ and $\langle t'^j: j < \omega\rangle$ and an increasing sequence of integers $\langle l^j: j < \omega\rangle$ such that $s \cup \tau^j$ forces t^j, $s' \cup \tau^j$ forces t'^j and $t^j(l^j) \neq t'^j(l^j)$. But then let $x = s \cup \bigcup\{\tau^j: j < \omega\}$ and $x' = s' \cup \bigcup\{\tau^j: j < \omega\}$.

We have $x =_* x'$ while $\varphi(x) \neq_* \varphi(x')$, a contradiction. The claim is proved.

To ensure (ii) we apply the claim repeatedly for each pair $\langle s, s'\rangle \in 2^{n_i} \times 2^{n_i}$.

Enumerate $2^{n_i} \times 2^{n_i}$ as $\{\langle s^j, s'^j\rangle: j < 2^{2n_i}\}$ and construct $\langle g_i^j: j \leqslant 2^{2n_i}\rangle$ and $\langle n_i^j: j \leqslant 2^{2n_i}\rangle$ by induction. Set $g_i^0 = g_i$ and $n_i^0 = n_i$. Given g_i^j and n_i^j find $n_i^{j+1} > n_i^j$ and g_i^{j+1} using the claim such that $g_i^{j+1} \supseteq g_i^j$, g_i^{j+1}: $[n_i, n_i^{j+1}) \to 2$ and g_i^{j+1} is a witness for $\langle s^j, s'^j\rangle$. Finally, we let $f_{i+1} = g^{2^{2n_i}}$ and $n_i = n_i^{2^{2n_i}}$. This finishes the inductive construction of the f_i's and n_i's.

Now, let $a^\varepsilon = \bigcup\{[n_i, n_{i+1}): i \equiv \varepsilon \bmod 3\}$ and $f^\varepsilon = \bigcup\{f_i: i \equiv \varepsilon \bmod 3\}$ for $\varepsilon = 0, 1, 2$. Then consider the function $\varphi^0(x) = \varphi(x \cup a^1 \cup a^2) \setminus \varphi(a^1 \cup a^2)$. Since F is a homomorphism we have $\forall x \subseteq a^0$, $\varphi^0(x) =_* \varphi(x)$. It is easy to see that there are functions h_i such that

(v) h_i: $\mathscr{P}([n_{3i}, n_{3i+1})) \to \mathscr{P}([n_{3i-1}, n_{3i+2}))$ for $i < \omega$,

(vi) $\varphi^0(x) = \bigcup\{h_i(x \cap [n_{3i}, n_{3i+1})): i < \omega\}$ for $x \subseteq a^0$.

Now, for almost all $i < \omega$ we must have

(vii) $\forall u, v \in \mathscr{P}([n_{3i}, n_{3i+1}))$, $h_i(u \cup v) = h_i(u) \cup h_i(v)$.

Assume there is an infinite $A \subseteq \omega$ such that for every $i \in A$ there are u_i, $v_i \subseteq [n_{3i}, n_{3i+1})$ and $h_i(u_i \cup v_i) \neq h_i(u_i) \cup h_i(v_i)$. Then let $u = \bigcup\{u_i: i \in A\}$ and $v = \bigcup\{v_i: i \in A\}$. Then $\varphi(u \cup v) \neq_* \varphi(u) \cup \varphi(v)$ which contradicts the fact that F is a homomorphism. Similarly we get that h_i maps singletons to singletons for almost all $i < \omega$. For, assume otherwise. Let $A \subseteq \omega$ be infinite and $l_i \in [n_i, n_{i+1})$ for $i \in A$ such that $|h_i(\{l_i\})| \geqslant 2$ for every $i \in A$. Pick $m_i \in h_i(\{l_i\})$ for $i \in A$ and let $y = \{m_i: i \in A\}$. Then, clearly, there is no x such that $\varphi(x) =_* y$. This contradicts the fact that F is onto.

So we can define a function h^0: $a^0 \to \omega$ by

$$h^0(l) = k \quad \text{if } l \in [n_{3i}, n_{3i+1}) \quad \text{and} \quad g_i(\{l\}) = \{k\}.$$

Then h^0 induces F on $\mathscr{P}(a^0)/\text{fin}$. Similarly we can find h^1 and h^2 which induce F on $\mathscr{P}(a^1)/\text{fin}$ and $\mathscr{P}(a^2)/\text{fin}$ respectively. By combining them and changing on a finite set if necessary we obtain the desired conclusion.

Let us now consider the general case, that the function φ is defined only on a dense G_δ set X. So there are open dense sets U_n, $n < \omega$ such that $X = \bigcap_{n<\omega} U_n$. We construct by induction an increasing sequence of integers $\langle m_k: k < \omega\rangle$ and a sequence of finite functions $\langle f_k: k < \omega\rangle$ such that f_k: $[m_k, m_{k+1}) \to 2$ and, for every $s \in 2^{m_k}$, $N_{s \cup f_k} \subseteq U_k$. Then let $f^\varepsilon = \bigcup\{f_k: k \equiv \varepsilon \bmod 2\}$ and $a^\varepsilon = \bigcup\{[m_k, m_{k+1}): k \equiv \varepsilon \bmod 2\}$ for $\varepsilon = 0, 1$. Then for every $x \subseteq a^\varepsilon$, $x \cup f^{1-\varepsilon} \in X$ so we can define φ^ε: $\mathscr{P}(a^\varepsilon) \to 2$ by $\varphi^\varepsilon(x) = \varphi(x \cup f^{1-\varepsilon}) \setminus \varphi(f^{1-\varepsilon})$. Clearly φ^ε is continuous for $\varepsilon = 0, 1$. So, by the special case of the lemma we get h^0, h^1 which induce F on $\mathscr{P}(a^0)$ and $\mathscr{P}(a^1)$ respectively. But then by combining them we conclude that F is trivial. □

The above lemma clearly implies that if $F \in \text{Aut}(\mathscr{P}(\omega)/\text{fin})$ and there is a Baire choice function for F, then F is trivial. We also have the corresponding result for a Lebesgue measurable choice function.

Proposition 3. *Let $F \in \text{Aut}(\mathscr{P}(\omega)/\text{fin})$. Suppose there is a set X of positive measure and a Lebesgue measurable function φ: $X \to 2^\omega$ such that $\forall x \in X$ $F[x] = [\varphi(x)]$. Then F is trivial.*

PROOF. By Luzin's theorem (see [8, p. 53]) there is a compact set C of positive measure such that $\varphi \upharpoonright C$ is continuous. Let $s \in 2^{<\omega}$ be such that $\mu(C \cap N_s) > \frac{1}{2}\mu(N_s)$. By changing φ if necessary we may assume s is a sequence of 0's of length $n < \omega$. Consider the addition modulo 2, i.e. $x + y$ is such that $\forall k < \omega$, $x(k) + y(k) \equiv z(k) \bmod 2$ for $x, y \in 2^{\omega}$, $z \in 2^{\omega}$.

Claim. $N_s \subseteq C + C = \{x + y\colon x, y \in C\}$.

PROOF. Fix $z \in N_s$. Then $z + (C \cap N_s) \subseteq N_s$ and $\mu(z + (C \cap N_s)) > \frac{1}{2}\mu(N_s)$. So, $(z + C) \cap C \neq \varnothing$. Pick $x, y \in C$ such that $z + x = y$. But then also $z = x + y$.

Now, consider the lexicographical ordering of 2^{ω}. Define

$$h(z) = \min((z + C) \cap C)$$

and $g(z) = z + h(z)$ for $z \in N_s$. Finally, let $\psi(z) = \varphi(h(z)) + \varphi(g(z))$. Then ψ is a Baire function, $\psi\colon N_s \to 2^{\omega}$ and $\forall z \in N_s$, $F[z] = [\psi(z)]$. This is sufficient to conclude, by virtue of the previous lemma, that F is trivial. □

COROLLARY 4. *Every Borel automorphism of* $\mathscr{P}(\omega)/\mathrm{fin}$ *is trivial.*

PROOF. Let F be a Borel automorphism. So $\mathrm{Gr}(F)$ is a Borel subset of $2^{\omega} \times 2^{\omega}$ with countable sections. But such sets can be uniformized by Borel sets (see Moschovakis [6, p. 258]). Hence there is a function $\varphi\colon 2^{\omega} \to 2^{\omega}$ such that $\langle x, \varphi(x)\rangle \in \mathrm{Gr}(F)$ for every $x \in 2^{\omega}$ and $\mathrm{Gr}(\varphi)$ is Borel. But it is well known that such φ is continuous on a dense G_{δ} set, so the result follows from Lemma 2. □

Clearly, the above result is true for analytic automorphisms, too, since they are in fact Borel.

COROLLARY 5. *Axiom of Determinacy implies that every automorphism of* $\mathscr{P}(\omega)/\mathrm{fin}$ *is trivial.*

PROOF. AD implies that every $G \subseteq 2^{\omega} \times 2^{\omega}$ with nonempty sections can be uniformized by a continuous function on a dense G_{δ} set. This is a well-known fact, first proved by Solovay, but let us sketch the argument for the interested reader. Consider the following infinite game played by two players, I and II.

I	σ_0		σ_1		σ_2		$\cdots$
II		τ_0		τ_1		τ_2	$\cdots$
		n_0		n_1		n_2	

I plays finite partial functions σ_i (i.e. elements of $2^{<\omega}$) while II plays finite partial functions τ_i and integers n_i. In the end they produce two sequences $x = \sigma_0 \frown \tau_0 \frown \sigma_1 \frown \tau_1 \frown \cdots$ and $y = \langle n_0, n_1, \ldots\rangle$. We declare that II wins iff $\langle x, y\rangle \in G$.

A winning strategy for II easily gives rise to a uniformizing continuous function on a dense G_{δ} set. By AD we will be done once we prove the following.

Claim. I *does not have a winning strategy.*

PROOF. Fix a strategy S for I. Enumerate $2^{<\omega}$ as $\{s_i\colon i < \omega\}$ in such a way that $s_i \subseteq s_j$ implies $i \leqslant j$. We build by induction an increasing sequence of elements of $2^{<\omega}$, $\{q_i\colon i < \omega\}$ and a sequence of partial plays $\{p_i\colon i < \omega\}$ such that

(i) p_i is a partial play according to S of the form

$$\begin{array}{ccccccc} \sigma_0^i & & \sigma_1^i & & \cdots & & \sigma_k^i \\ & \tau_0^i & & \tau_1^i & \cdots & \tau_{k-1}^i & \\ & s_i(0) & & s_i(1) & & s_i(k-1) & \end{array}$$

where $k = \text{length}(s_i)$ and such that $\sigma_0^i \frown \tau_0^i \frown \cdots \frown \sigma_k^i = q_i$.

(ii) If s_j extends s_i then p_j extends p_i.

We proceed by induction. Suppose we have built p_i and q_i for $i \leqslant n$. We construct p_{n+1} and q_{n+1}. First, look at s_{n+1}. Let $\text{length}(s_{n+1}) = k + 1$ and let $i \leqslant n$ be such that $s_i = s_{n+1} \restriction k$. Consider the partial play p_i. let $\tau_j^{n+1} = \tau_j^i$ for $j \leqslant k - 1$ and $\sigma_j^{n+1} = \sigma_j^i$ for $j \leqslant k$. Then let τ_k^{n+1} be such that $q_i \frown \tau_k^{n+1} = q_n$ and $\sigma_{k+1}^{n+1} = S(p_i \frown \langle \tau_k^{n+1}, s_{n+1}(k) \rangle)$. Finally put $p_{n+1} = p_i \frown \langle \tau_k^{n+1}, s_{n+1}(k) \rangle \frown \sigma_{k+1}^{n+1}$ and $q_{n+1} = q_n \frown \sigma_{k+1}^{n+1}$. This finishes the inductive construction of the p_i's and q_i's.

Now, let $x = \bigcup\{q_i\colon i < \omega\}$. It is clear that for every $y \in 2^\omega$ there is a run of the game p in which I uses his strategy S

$$\begin{array}{ccccccc} \sigma_0 & & \sigma_1 & \cdots & \sigma_n & & \cdots \\ & \tau_0 & & \tau_1 \quad \cdots & & \tau_n & \cdots \\ & y(0) & & y(1) & & y(n) & \end{array}$$

such that $\sigma_0 \frown \tau_0 \frown \sigma_1 \frown \tau_1 \frown \cdots \frown \sigma_n \frown \tau_n \frown \cdots = x$. Namely, if $y \restriction n = s_{i_n}$ we know that p_{i_m} extends p_{i_n} for $n < m$, so we can take $p = \bigcup\{p_{i_n}\colon n < \omega\}$. If y is chosen such that $\langle x, y \rangle \in G$, then II wins this run of the game thus showing that S is not a winning strategy for I. □

Our next theorem shows that Borel cannot be replaced by projective (in ZFC) in Corollary 2. However we do not know what happens with automorphisms which are between $\boldsymbol{\Sigma}_1^1$ and $\boldsymbol{\Delta}_2^1$ in complexity.

THEOREM 6. *The Axiom of Constructibility implies that there is a nontrivial* Δ_2^1 *automorphism of* $\mathscr{P}(\omega)/\text{fin}$.

PROOF. This can be proved by simply modifying Baumgartner's argument mentioned at the beginning of this paper, constructing the sequences $\{a_\alpha\colon \alpha < \omega_1\}$ and $\{f_\alpha\colon \alpha < \omega_1\}$ in the canonical way using a fixed good Δ_2^1-well ordering of $\mathscr{P}(\omega)$ (see Devlin [**1**, p. 70]). The details are straightforward and are left to the reader. □

We finally show that it is consistent with CH that all projective, in fact, ODR automorphisms are trivial. We use the well-known model of Solovay [**1**], see also the presentation in Jech [**2**, p. 537].

Let κ be an infinite cardinal. The Levy collapse $\mathscr{L}_\kappa$ is the following poset. Conditions are finite functions p, such that $\text{dom}(p) \subseteq \kappa \times \omega$ and $p(\alpha, n) \in \alpha$ for $(\alpha, n) \in \text{dom}(p)$. Order is reverse inclusion. Assuming that κ is inaccessible, Solovay [**11**] showed that the following holds in $V^{\mathscr{L}_\kappa}$.

Fact. Every ODR *subset of* $2^\omega \times 2^\omega$ *with nonempty sections can be uniformized on a dense* G_δ *set by a continuous function.*

By Lemma 2 this immediately yields the following.

THEOREM 7. *If the theory* "ZF + $\exists$ *inaccessible cardinal*" *is consistent, then so is the theory* "ZFC + CH + *every* ODR *automorphism of* $\mathscr{P}(\omega)/\text{fin}$ *is trivial*".

REMARKS. (1) Shelah **[10]** has shown that the above uniformization result can be proved consistent without using an inaccessible cardinal. So, for Theorem 7 we need only assume that ZF is consistent.

(2) It follows from a recent result of Foreman, Magidor, Shelah and Woodin that if there is a supercompact cardinal (in fact much less) every automorphism of $\mathscr{P}(\omega)/\text{fin}$ which is in $L[\mathscr{P}(\omega)]$ is trivial.

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