Set Theoretic Aspects of the Control Measure Problem.

We survey the history and present some recent results on the well known Control Measure Problem and its connections with Set Theory. The origin of the problem is due to Von Neumann in the 1930s who asked whether certain combinatorial properties of measure algebras are sufficient to characterise them and its final form was stated by Maharam in the early 1940s. Maharam defined the concept of a continuous exhaustive submeasure and asked if every such submeasure is equivalent to a measure. This problem is significant to Set Theory as it is related to a fundamental problem of Prikry who asked whether Cohen and random forcing are the only basic forcing notions with the countable chain condition. While these problems are still open, recently there has been some important progress in this area.