

# On separation question for tree languages

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A classical result due to Lusin says that two disjoint analytic sets can be always separated by a Borel set, but this property fails in general for co-analytic sets. A similar property has been well studied for all classical hierarchies in descriptive set theory.

We embark on investigation of the separation question for the hierarchy of sets of trees definable by alternating tree automata induced by the Rabin-Mostowski index or, equivalently, by the alternation of the least and greatest fixed points in the  $\mu$ -calculus.

A topological complexity turns out to be relevant here. In an analogy with the Lusin Theorem, any two disjoint Büchi recognizable sets can be always separated by a weakly recognizable set (essentially due to Rabin), but this property fails in general for co-Büchi sets. Indeed, we exhibit a pair of disjoint co-Büchi sets inseparable even by a Borel set, hence a fortiori by a weakly recognizable set.

We are now able to shift the negative result to the 3rd level of the hierarchy : we exhibit a pair of disjoint (1,3)-recognizable sets inseparable by any set of the corresponding Delta-level. The key property is that the (1,3)-recognizable sets of trees turn out to be analytically inductive (i.e., presentable as least fixed points of analytically definable operators). We use a game-theoretic representation of this class, due to Moschovakis, whereas the idea of our inseparable pair is inspired by the work by Saint Raymond.